Technical Notes

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Traveling-Wave-Type Flutter of Infinite Elastic Plates

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A TRAVELING-WAVE motion of the form

$$w(x,t) = Re\{\hat{w}e^{-ik(x-\hat{c}t)}\}, \qquad k|\hat{w}| \ll 1 \tag{1}$$

of an infinite plate, with the upper surface $z > 0$ exposed to an inviscid compressible flow in the positive x direction at a constant velocity U (Fig. 1), is governed by the following equation[1,2]

$$\hat{c}_0^2 - \hat{c}^2 + i(b\hat{c}/mk) - (\rho/mk\beta)(U - \hat{c})^2 = 0 \tag{2}$$

where

$$\beta = \{1 - [(U - \hat{c})a]^2\}^{1/2}, \qquad Re\beta > 0$$

Here k is the wave number (real and positive), $\hat{c}$ and $\hat{c}_0$ are the phase velocity of transverse waves along the panel in the flow and in vacuum, respectively, m is the mass density of the panel per unit area, ρ and a are the mass density of, and the sound velocity in, the undisturbed air, and b is the coefficient of the damping forces $b\partial w/\partial t$.

The air below the plate is at rest, and its pressure p_L is assumed to equal approximately p_∞, which is the pressure of the undisturbed air in the upper half space (Fig. 1).

The phase velocity $\hat{c}$ is generally a complex number. It is the eigenvalue to be determined from the equation of motion (2). Negative imaginary values of $\hat{c}$ imply instability of the assumed disturbance [Eq. (1)].

A detailed analysis of Eq. (2) for incompressible flow leads to the conclusion[2,3] that, while in the undamped system a flutter-type instability sets in at some critical velocity $U = U_F$, the damping, however small, reduces the critical velocity to $U_D < U_F$, and changes the character of the instability to a divergence type. In the region $U_D < U < U_F$ the instability is very weak if damping is small. It is the aim of this Note to explain how far these conclusions can be extended to compressible flow.

Introducing nondimensional variables $c = \hat{c}/a$, $c_0 = \hat{c}_0/a$, $M = U/a$, $\mu = \rho/mk$, $g = b/2mk\hat{c}_0$, Eq. (2) becomes

$$c_0^2 - c^2 - \mu(M - c)^2[1 - (M - c)^2]^{-1/2} + i2gcc_0 = 0 \tag{3}$$

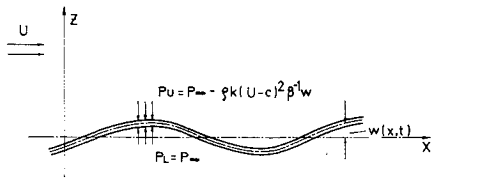

Fig. 1 Geometry of panel and notation.

Equation (3) with damping absent was thoroughly investigated by Miles,[1] who showed that only subsonic disturbances are possible, i.e., that always†

$$(M - c_R)^2 < 1 \tag{4}$$

and obtained an analytical (albeit parametric) expression for the neutral stability-instability boundary.

When damping is present it is convenient to substitute $c = c_R + ic_i$ in Eq. (3) and separating real and imaginary parts, one obtains respectively

$$c_0^2 - c_R^2 + c_i^2 - [\mu/(\beta_R^2 + \beta_i^2)][\beta_R(M_1^2 - c_i^2) - 2\beta_i c_i M_1] - 2gc_i c_0 = 0 \tag{5a}$$

$$2c_R c_i - [\mu/(\beta_R^2 + \beta_i^2)][\beta_i(M_1^2 - c_i^2) + 2\beta_R c_i M_1] + 2gc_R c_0 = 0 \tag{5b}$$

where

$$M_1 = M - c_R, \quad \beta_R = (M_1 c_i/\beta_i)$$
$$2\beta_i^2 = [(1 + c_i^2 - M_1^2)^2 + 4c_i^2 M_1^2]^{1/2} - (1 + c_i^2 - M_1^2) \tag{6}$$

and the sign of β_i should be chosen so as to make β_R positive in accordance with Eq. (2).

Since in the undamped system c_i vanishes below the critical Mach number, we assume that, by continuity, for very small values of g, c_i is also very small. On this assumption, Eqs. (6) can be rewritten approximately so, for

$$c_i^2 \to 0_+: \quad \beta_i \simeq [c_i M_1/(1 - M_1^2)^{1/2}], \quad \beta_R \simeq (1 - M_1^2)^{1/2} \tag{7}$$

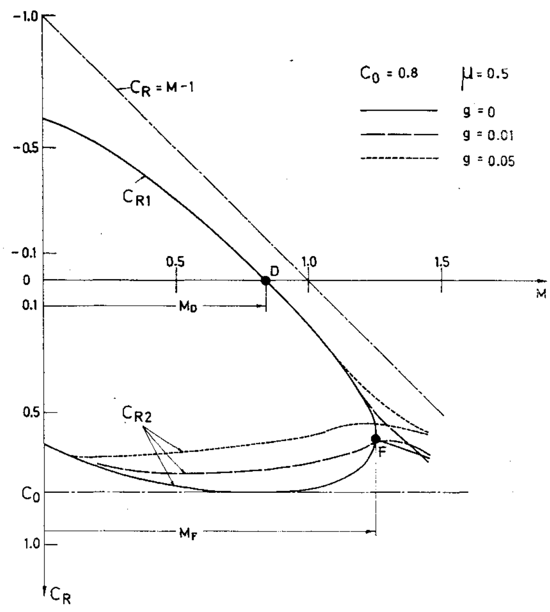

Fig. 2 Wave speed (real part) vs Mach number.

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† Another restriction imposed on c_R is $c_R^2 \leq c_0^2$ since μ in Eq. (3) is nonnegative.

and on substitution of Eq. (7) in Eq. (5) one obtains‡

$$c_0^2 - c_R^2 - \mu(M - c_R)^2/[1 - (M - c_R)^2]^{1/2} - 2gc_ic_0 \simeq 0, \qquad (8a)$$

$$gc_Rc_0 - c_Rc_i[1 - \mu\{(M - c_R)/c_R[1 - (M - c_R)^2]^{1/2}\} \times (1 + \{(M - c_R)^2/2[1 - (M - c_R)^2]\})] = 0 \qquad (8b)$$

Comparison of Eqs. (8a) and (3) reveals that for very small values of g the c_R's are practically unaffected by damping, while Eq. (8b) implies that c_i can be negative only when the ratio $(M - c_R)/c_R$ is large and positive;

$$(M - c_R)/c_R \gg 0 \qquad (9)$$

To gain better understanding of the phenomenon, and to check when requirement (9) becomes satisfied, the complex phase velocity c should be presented as a continuous function of the Mach number M in at least some typical cases.

Keeping c_0 fixed and varying M, both c_R and c_i satisfying Eqs. (4) and (5) were computed for a wide range of mass ratios μ. Typical results§ for $c_0 = 0.8$ and $\mu = 0.5$ plotted in Figs. 2, and 3 show that in the undamped system at low flow velocities two distinct real roots exist, $c_{R1} < 0$ and $c_{R2} > 0$. The effect of the flow consists in drawing them closer until they merge at some critical Mach number $M = M_F$, achieving a positive value $c_{R1} = c_{R2} = (c_R)_{cr}$ (point F in Fig. 2). Henceforth, the roots become complex conjugate, with c_i's rising very steeply (Fig. 3), thus indicating a severe instability of one of the waves.

When damping is present the real parts c_R of the roots do not change considerably (Fig. 2) in accordance with the previous analysis of Eq. (8a). It should be pointed out that the roots no longer merge. The behavior of the imaginary parts c_i of the roots is, however, completely different. Both c_{i1} and c_{i2} are positive from the very beginning, indicating damped motion, up to $M = M_D$ (Figs. 3 and 2) at which $c_{R1} = c_{i1} = 0$. Henceforth, c_{i1} becomes negative, decreasing steadily although initially very slowly, implying the onset of a weak divergence type instability at $M = M_D$, which is lower than the flutter boundary M_F of the undamped system. The second root c_{i2} remains always positive.

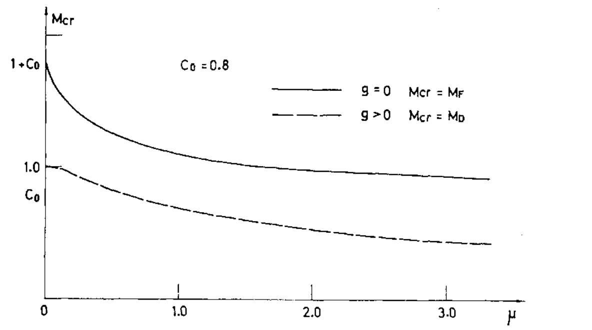

Fig. 4 Stability boundaries of undamped and damped systems.

A glance at Fig. 2 reveals that the ratio $(M - c_R)/c_R$ is initially negative and the requirement (9) is satisfied by c_{R1} only when the latter changes its sign. This confirms the preceding results obtained analytically while discussing Eq. (8b).

To find the magnitude of M_D we substitute $c_R = 0$ in Eq. (5b) obtaining $c_i = 0$ whence by Eq. (5a)

$$c_0^2(1 - M_D^2)^{1/2} = \mu M_D^2$$

or (in terms of Miles's[1] variables)

$$(c_0/U)_{cr}^2 = \mu(1 - M^2)^{-1/2} \qquad (10a)$$

or explicitly

$$M_D^2 = \tfrac{1}{2}(c_0^2/\mu)^2\{[1 + 4(\mu/c_0^2)^2]^{1/2} - 1\} \qquad (10b)$$

A summary plot of flutter and divergence boundaries for the case $c_0 = 0.8$ and various mass ratios is presented in Figs. 4 and 5 for the undamped and damped systems, respectively. The destabilizing effect of damping is evident.

It should be emphasized that M_D, determined by Eqs. (10), is independent of the damping ratio g and always smaller than unity, thus indicating that in the presence even of negligible damping every panel is unstable in supersonic flow (compare Refs. 4, 5).

Summing up, we conclude that the destabilizing effect of viscous damping in subsonic flow is qualitatively similar to that of incompressible flow. In supersonic flow, every panel is rendered unstable. The (damping induced) instability is of a divergence type, but is very mild in slightly damped systems.

Strictly speaking, from the point of view of Liapunov's theory of stability, the concept of "damping induced instability" is not very appropriate. The critical flow speed U_F determined on the assumption of no damping corresponds to the doubtful case in the Liapunov's theory and should be referred to "quasicritical."[6]

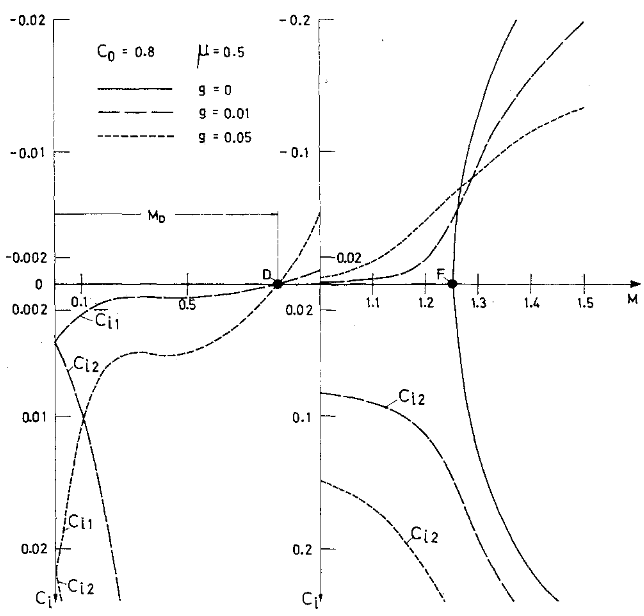

Fig. 3 Wave speed (imaginary part) vs Mach number. Note the difference in scales for $M > 1$ and $M < 1$.

‡ In passing from Eqs. (5) to (8) we assumed, strictly speaking, that c_i^2 is much smaller than $(1 - M_1^2)$, $(1 - M_1^2)M_1^{-2}$, and $|c_0^2 - c_R^2|$; however, Eqs. (8) seem to be valid under much weaker restrictions.

§ Similar, although less complete, results are published in Ref. 4 dealing with a closely related problem.

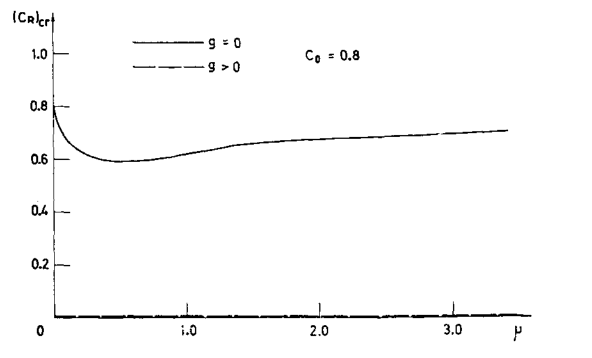

Fig. 5 Wavespeed at stability boundaries.

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Dynamical Equations of Nonrigid Satellites

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Nomenclature

dV = differential volume element
$\tilde{\mathbf{F}}$ = force per unit mass
$\mathbf{F}$ = total external force on satellite
$\mathbf{G}^m$ = total torque on m about its c.m.
$\mathbf{H}^m$ = angular momentum of m about its c.m.
$[\mathbf{J}^m]$ = inertia dyadic of m about its c.m.
M = mass of entire satellite; also indicates the entire satellite
M^m = mass of satellite main body
m = satellite main body
o = origin of body frame b
$\mathbf{R},\mathbf{R}^m$ = vector from o to c.m. of M and m, respectively
$\mathbf{r}$ = vector from o to dV
$\mathbf{V}$ = absolute translational velocity of the c.m. of M
$\mathbf{y},\mathbf{y}^m$ = vector to dV from c.m. of M and m, respectively
$\mathbf{Z}$ = absolute acceleration of $\mathbf{R}$ with $\ddot{q}_j$ and $\dot{\boldsymbol{\omega}}$ components deleted
ρ = density
$\boldsymbol{\omega}$ = absolute angular velocity of frame b
[ ] = dyadic

Introduction

RECENTLY published techniques enable dynamical equations of deformable satellites to be derived directly and efficiently when the satellite is represented as a set of rigid bodies joined at hinge points in a tree topology.[1,2] The purpose of the present Note is to indicate an analogous method which is applicable when satellite nonrigidity is modeled by generalized coordinates. The approach enables a direct development of dynamical equations that can include configuration changes and large deformations, angular rates, and attitude motions. The dynamical equations of such systems commonly are derived by a formal application of Lagrange's equation. The study by Farrell and Newton[3] is an excellent example of the formal use of Lagrange's equation in a difficult nonrigid satellite problem. The Lagrangian approach, however, has several disadvantages. The manual labor needed to derive and differentiate the kinetic energy expression can be time consuming and difficult to accomplish without error. The resulting equations are difficult to modify after they have been developed, and the significance of individual terms often is obscure. In satellite problems, the resulting rotation equations often are less convenient than those obtained by angular momentum techniques. The generalized forces of rotation are not merely torque vector components resolved on an orthogonal frame, and the possibility of simplifying the rotation equations by using internal torques is precluded. Assuming that attitude is specified by Euler angles, the dynamics equations are made cumbersome by the presence of trigonometric functions of these angles.

The aforementioned disadvantages in the use of Lagrange's equation are alleviated by the technique described here. Two main equations are presented. The first defines the satellite dynamics relative to a frame b which is attached to the satellite in an arbitrary manner. The second defines frame b's rotational kinetics. Translation of the c.m. is not treated, because the equations are well known. The relative motion equation essentially is equivalent to an equation given by Frazer, Duncan, and Collar,[4] whereas the rotation equation essentially is equivalent to a result attributed to Liouville.[5] The advantage of the equations of the present Note over those which have appeared previously is that the individual terms have been expressed as explicitly as possible without a sacrifice in generality using generalized coordinates and their derivatives. This enables dynamical equations of nonrigid satellites to be developed in an efficient manner with a reduced probability of algebraic or conceptual errors.

Equations used by Buckens[6] and Ashley[7] are special cases of those derived here. Both these authors employed normal modes and a linearized deformation model. Neither was specifically concerned with developing general nonlinear dynamical equations of nonrigid satellites. The "hybrid coordinate" approach used by Likins and Wirsching[8] is a special case of the method of separating system dynamics into relative and rotational degrees of freedom employed in the present work. Reference 8 was concerned mainly with the efficiency of the linearized dynamical equations when used in a computer simulation; emphasis was on simplifying the inertia matrix. The present study does give some consideration to the inertia matrix problem. The main interest, however, is on reducing the effort needed to derive nonlinear equations.

The final equations are Eqs. (3–7, and 10–11). The variables q, which define the satellite state relative to frame b, have been separated into two sets: 1) a set indicated by subscripts i and $j(i,j = 1 \text{ to } n)$ whose dynamics are obtained from the kinetics equations and 2) a set indicated by subscript $\alpha(\alpha = n + 1 \text{ to } N)$ whose time responses are known a priori or established by a control law. Subscripts u and $v(u,v = 1 \text{ to } N)$ indicate the complete q set. Use of the same subscript twice in a product of terms indicates a summation over the range of the indexes. A comma is used in the conventional manner to indicate differentiation. Dyadic brackets [ ] around a vector indicate the usual skew-symmetric form.

In the rotation equations, the satellite is represented as a main body m to which auxiliary bodies s are attached. Neither m nor s are assumed to be rigid. s consists of those bodies whose interaction torques $\mathbf{G}^s$ and forces $\mathbf{F}^s$ on m are readily computable. When applicable, the s technique has the potentiality of simplifying the rotation equations. In some problems, certain components of $\mathbf{F}^s$ and $\mathbf{G}^s$ can be obtained only from the external loading, including the inertial forces, on s. An example is a satellite with a main body m and booms s whose bending deformations are defined by

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